

TIME TABLE

| TIME | Monday July 16 | Tuesday July 17 | Wednesday July 18 | Thursday July 19 | Friday July 20 |
|---------------|-------------------|--------------------|----------------------|---------------------|-------------------|
| 9.00 - 9.45 | Registration | Gassner | Rebay | Löhner | Fernandez-M |
| 9.45 - 10.30 | Kronbichler | Gassner | Rebay | Löhner | Fernandez-M |
| 11.00 - 11.45 | Kronbichler | Kronbichler | Gassner | Fernandez-M | Löhner |
| 11.45 - 12.30 | Kronbichler | Kronbichler | Gassner | Fernandez-M | Löhner |
| 14.00 - 14.45 | Rebay | Persson | Kronbichler | Fernandez-M | |
| 14.45 - 15.30 | Rebay | Persson | Kronbichler | Fernandez-M | |
| 16.00 - 16.45 | Gassner | Rebay | Persson | Persson | |
| 16.45 - 17.30 | Gassner | Rebay | Persson | Persson | |
| 18.00 | Welcome Aperitif | | | | |

ADMISSION AND ACCOMMODATION

The registration fee is 600.00 Euro + VAT*, where applicable (bank charges are not included). The registration fee includes a complimentary bag, four fixed menu buffet lunches (on Friday upon request), hot beverages, downloadable lecture notes and wi-fi internet access.

Applicants must apply at least one month before the beginning of the course. Application forms should be sent on-line through the following web site: <http://www.cism.it>. A message of confirmation will be sent to accepted participants. Applicants requiring assistance with the registration should contact the secretariat at the following email address cism@cism.it.

Applicants may cancel their course registration and receive a full refund by notifying CISM Secretariat in writing (by email to cism@cism.it) no later than two weeks prior to the start of the course.

Cancellation requests received during the two weeks prior to the start of the course will be charged a 50.00 Euro handling fee. Incorrect payments are also subject to a 50.00 Euro handling fee.

A limited number of participants from universities and research centres who are not supported by their own institutions can be offered lodging and/or board, if available, in a reasonably priced hotel or student guest house.

Requests should be sent to CISM Secretariat by **May 16, 2018** along with the applicant's curriculum and a letter of recommendation by the head of the department or a supervisor confirming that the institute cannot provide funding. Preference will be given to applicants from countries that sponsor CISM.

Information about travel and accommodation is available on the web site www.cism.it, or can be mailed upon request.

* Italian VAT is 22%.

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EFFICIENT HIGH-ORDER DISCRETIZATIONS FOR COMPUTATIONAL FLUID DYNAMICS

CISM-ECCOMAS International Summer School
coordinated by

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University of California, Berkeley
USA



Udine July 16 - 20 2018

EFFICIENT HIGH-ORDER DISCRETIZATIONS FOR COMPUTATIONAL FLUID DYNAMICS

This course introduces modern high-order methods for computational fluid dynamics. As compared to low order finite volumes predominant in today's production codes, higher order discretizations significantly reduce dispersion errors, the main source of error in long-time simulations of flows at higher Reynolds numbers. Thus, they make previously intractable problems accessible to simulation in an increasingly wide range of applications. However, a careful selection of algorithms and implementations is fundamental to fully unleash the potential of high order schemes, in particular for emerging high-performance computer architectures that approach the exascale threshold. A major goal of this course is to teach the basics of the discontinuous Galerkin (DG) method in terms of its finite volume and finite element ingredients. Consistent numerical fluxes over

the element boundaries take directionality of flow into account and introduce some numerical dissipation. High-order shape functions inside the element provide accurate approximations and geometric flexibility. This special combination makes DG not only high-order convergent but also robust for transport-dominated problems. In the presentation of the method, favorable numerical fluxes and recent developments regarding the particular arrangement of the weak form for turbulent flows are discussed. Furthermore, implementation techniques that have their origin in the spectral element community will be presented. These so-called sum factorization kernels avoid building a global Jacobian matrix and instead evaluate differential operators by particular fast integration schemes for tensor product shape functions

and quadrature formulas. The complexity of the resulting DG operator evaluation is competitive with finite differences without compromising the geometric flexibility and robustness, as showcased by a compressible flow solver with explicit time integration. Sum factorization is also increasingly used in implicit scenarios, for example for solving the pressure Poisson equation in splitting schemes for the incompressible Navier-Stokes equations with multigrid solvers. An alternative technique that makes matrix-based DG competitive is the hybridizable discontinuous Galerkin (HDG), where the numerical fluxes are expressed in terms of a new variable on all faces, the mesh skeleton. The particular construction allows HDG to eliminate all the degrees of freedom inside the elements in favor of the variables on the mesh skeleton by a static-condensation-like approach,

considerably reducing the final linear system size. Together with improved convergence rates, efficiency gets a significant boost. The course also discusses the computational efficiency of high-order methods versus state-of-the-art low order methods in the finite difference context, given that accuracy requirements in engineering are often not overly strict. Thus, the faster convergence rates of high-order methods in the asymptotic regime must be put in a quantitative context, in particular with respect to the nonlinear interaction between scales typical for fluid dynamics. This comparative setup enables the participants to obtain a broader perspective on high-order methods and identify major challenges in the field for the next decade.

INVITED LECTURERS

Sonia Fernández-Méndez - Universitat Politècnica de Catalunya, Barcelona, Spain
6 lectures on: Hybridizable Discontinuous Galerkin (HDG) methods for the Laplace equation: implementation, revision of a Matlab code, convergence and superconvergence, efficiency comparison vs continuous FEM. HDG formulation for incompressible flow problems: implementation, Matlab code, efficiency and stability comparison vs continuous FEM.

Gregor Gassner - Köln University, Germany
6 lectures on: Split-form discontinuous Galerkin methods for compressible Fluid Dynamics, DG and summation-by-parts, discrete stability and de-aliasing, high order DG and explicit turbulence modeling.

Martin Kronbichler - Technical University of Munich, Germany
7 lectures on: Introduction to discontinuous Galerkin methods for scalar transport equations, sum factorization for evaluation of integrals in context of explicit time integration, efficient solution of linear systems with fast operator evaluation and its limitations. High-performance computing aspects in DG: parallel scalability and fast absolute performance on the next-generation exascale computers.

Rainald Löhner - George Mason University, Fairfax, VA, USA
4 lectures on: The tradeoff of high versus low order methods: What accuracy is relevant in an engineering application? What is the speed/accuracy ratio of high order methods? What happens to high order methods in applications where monotonicity is important? Are high order methods relevant for applications where butterfly effects are present?

Per-Olof Persson - University of California, Berkeley, USA
6 lectures on: Practical use of high-order discontinuous Galerkin methods for problems in fluid and solid mechanics: sparse discretizations, efficient preconditioners and iterative solvers, parallel implementations, calculations of sensitivities using fully discrete adjoints. Various methods for generation of appropriate curvilinear meshes. Specialized solvers for multiphysics problems such as fluid-structure interaction.

Stefano Rebay - Università degli studi di Brescia, Italy
6 lectures on: Accurate numerical fluxes for compressible and for incompressible flows: numerical fluxes in finite volume and DG approximations of hyperbolic problems, exact and approximate Riemann solvers, Riemann solvers and high-order accurate DG methods, application to incompressible flows, limitations related to specific problem physics (low mach numbers, low shock speeds, wall overheating, etc.).

LECTURES

All lectures will be given in English. Lecture notes can be downloaded from the CISM web site. Instructions will be sent to accepted participants.

PRELIMINARY SUGGESTED READINGS

D. Flad, G. J. Gassner. On the use of kinetic energy preserving DG-schemes for large eddy simulation. Preprint arXiv:1706.07601 (2017).

B. Froehle, P.-O. Persson. A high-order discontinuous Galerkin method for fluid-structure interaction with efficient implicit-explicit time stepping. *Journal of Computational Physics* 272:455-470 (2014).

G.J. Gassner, A.R. Winters and David A. Kopriva. Split Form Nodal Discontinuous Galerkin Schemes with Summation-By-Parts Property for the Compressible Euler Equations. *Journal of*

Computational Physics 327:39-66 (2016).

G. Giorgiani S. Fernández-Méndez, A. Huerta. Hybridizable discontinuous Galerkin with degree adaptivity for the incompressible Navier-Stokes equations. *Computers & Fluids* 98:196-208 (2014).

M. Kronbichler, W. A. Wall. A performance comparison of continuous and discontinuous Galerkin methods with fast multigrid solvers. Preprint arXiv:1611.03029 (2016).

R.J. LeVeque. *Numerical methods for conservation laws*, Springer.

Originally published by Birkhauser Verlag (1992).

R. Lohner. Improved Error and Work Estimates for High Order Elements. *International Journal on Numerical Methods in Fluids* 72(11):1207-1218 (2013).

R. Lohner, E. Haug, A. Michalski, D. Britto, A. Degro, R. Nanjundaiah and R. Zarfam. Recent Advances in Computational Wind Engineering and Fluid-Structure Interaction. *Journal of Wind Engineering and Industrial Aerodynamics* 144:14-23 (2015).

R. Sevilla and Antonio Huerta. *Tutorial on hybridizable*

discontinuous Galerkin (HDG) for second-order elliptic problems. In J. Schröder and P. Wriggers (eds.), *Advanced Finite Element Technologies*, volume 566 of CISM International Centre for Mechanical Science, pp. 105-129 (2016).

E.F. Toro. *Riemann solvers and numerical methods for fluid dynamics. A practical introduction*. Third edition, Springer (2009).

W. Pazner and P.-O. Persson, Approximate tensor-product preconditioners for very high order discontinuous Galerkin methods. reprint arXiv:1704.04549 (2017).